Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

A GRAPHIC SOLUTION OF THE CUBIC EQUATION.[1]

By J. P. Ballantine, University of Maine.

The purpose of this paper is to suggest a graphic method of finding the real roots of a numerical cubic equation, a method which will involve comparatively little work when there are a large number of problems.

Let us consider the general cubic reduced to the standard form

$$u^3 + u^2 + xu = y. \tag{1}$$

This can easily be effected by dividing by the coefficient of u^3, and then by dividing the roots of the equation by the coefficient of u^2. It may happen in a particular case that this latter coefficient is zero, in which case the form obtained will differ from the form given above in that the u^2 term is missing; but a similar discussion is easily worked out for that case.

All the cubics of the standard form are easily set in a one to one correspondence with the points of the plane. Let us say that the x and the y of the cubic are the coördinates of the corresponding point.

The necessary and sufficient condition that v be a root of the standard cubic is that:

$$v^3 + v^2 + xv = y. \tag{2}$$

The same is a necessary and sufficient condition that the point (x, y) lie on a certain straight line, of which the above is said to be the equation. The points therefore corresponding to a set of isoradical cubics[2] lie on a straight line, and the common root of the cubics is the slope of the line.

Let us next consider the locus of points corresponding to cubics with two equal roots. The necessary and sufficient condition that v be a double root of the standard cubic is that:

---

[1] While the essential features of this solution are not new they are probably sufficiently unfamiliar to readers of the MONTHLY to render Mr. Ballantine's presentation of his rediscovery of them a matter of general interest. As applied to the equation $u^3 + xu + y = 0$ there is a recent discussion in J. Lipka, *Graphical and Mechanical Computation*, 1918, pp. 35–36. A similar discussion is found in C. Runge, *Graphical Methods*, New York, 1912, pp. 59–60 (also *Praxis der Gleichungen*, Leipzig, 1900, p. 159), and L. E. Dickson incorrectly attributes the method to Runge (Dickson, *Elementary Theory of Equations*, New York, 1914, p. 17). This method originated with Léon Lalanne who made the discovery in 1843 and regarded the cubic as a special case of the trinomial equation $u^m + xu^n + y = 0$ ("Sur les tables graphiques et sur la géométrie anamorphique . . .," *Annales des ponts et chaussées*, 1846, 1er semestre, pp. 27–29; *Comptes rendus de l'académie d. Sc.*, Paris, tome 81, 1875, pp. 1187–1188; and "Méthodes graphiques pour l'expression des lois à trois variables," *Notices réunies par le ministere des travaux publics à l'occasion de l'Exposition universelle de Paris*, 1878). Lalanne found that his form of the cubic equation led to the semi-cubical parabola $4x^3 + 27y^2 = 0$ as an envelope.

The subject is also discussed in J. de la Gournerie, *Traité de Géométrie descriptive*, seconde édition, troisième partie, Texte, Paris, 1885, pp. 211–212, Atlas, 3, fig. 455 (filling a quarto page), plate 46; and in M. d'Ocagne, *Traité de Nomographie*, Paris, 1899, pp. 43–44.—EDITOR.

[2] Two cubics are said to be isoradical if they have a root in common. A straight line is said to be isoradical if its points correspond to isoradical cubics.

$$(3)\qquad \begin{cases} v^3 + v^2 + xv = y \\ 3v^2 + 2v + x = 0. \end{cases}$$

This is also the necessary and sufficient condition that the point (x, y) lie on the envelope of the straight line. Every isoradical straight line is therefore tangent to the locus

$$(4)\qquad 4x^3 - x^2 + 18xy + 27y^2 - 4y = 0$$

obtained by eliminating v from (3).

All that is necessary, therefore, in order to solve a cubic, is to reduce it to standard form, obtaining the quantities x and y. Plot the corresponding point, and drop the three tangents to the locus (4). The slopes of the three tangents will be the three roots of the equation as written in standard form.

Note that the locus (4) can be plotted once for all, and in place of plotting a cubic equation, one need only plot a point for each problem to be solved.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

Psychologists today are restoring to honor, albeit in much modified form, the Old Reliable Dream Book. Theologists and educators are seeking to read the future. It is not surprising that a claim should be made for the use of mathematics as a means of prediction. Such a claim is stated by Professor Weaver in the first discussion this month. His conception must not be dismissed as fantastic without an investigation into its meaning. He makes quite clear that no prediction of individual events or circumstances is intended; his hope is at most that the progress of history in the large or average sense may be forecasted. In support of his contention may be cited the service of mathematics in exactly this function of prediction in connection with the physical sciences. It may even be argued that an example of similar application to the social sciences is to be found in the generally accepted theory of the periodicity of economic cycles of prosperity and depression, with an approximate period of ten or eleven years. Against this argument may be placed the apparently hopeless complexity, as regards mathematical formulation, of the social problem, compared with the physical problem. It is doubtful whether the reduction to concise qualitative laws, or even the specification of the independent variables in terms of which such laws are to be stated lies within the mental potentialities of the human race.

Several details in Professor Weaver's account suggest comment. For example, it is not obvious that the analogy of the determination of all values of an everywhere analytic function by the values of the function and its derivatives